

# DCS/CSCI 2350: Social & Economic Networks

*What can we say about a  
network having friends as well as  
enemies?*

**Structural Balance**  
Reading: Ch 5 of EK

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## World War I

The New York Times

The Opinion Pages

### Opinionator

OPINIONATOR

#### The Enemy of My Enemy

By STEVEN STROGATZ FEBRUARY 14, 2010 5:30 PM

It's traditional to teach kids subtraction right after addition. That makes sense — the same facts about numbers get used in both, though in reverse. And the black art of "borrowing," so crucial to successful subtraction, is only a little more baroque than that of "carrying," its counterpart for addition. If you can cope with calculating  $23 + 9$ , you'll be ready for  $23 - 9$  soon enough.

At a deeper level, however, subtraction raises a much more disturbing issue, one that never arises with addition. Subtraction can generate negative numbers. If I try to take 6 cookies away from you but you only have 2, I can't do it — except in my mind, where you now have negative 4 cookies, whatever that means.

Subtraction forces us to expand our conception of what numbers are. Negative numbers are a lot more abstract than positive numbers — you can't see negative 4 cookies and certainly can't eat them — but you can think about them, and you *have* to, in all aspects of daily life, from debts and overdrafts to contending with freezing temperatures and parking garages.

MORE IN THIS SERIES

- Rock Groups (Feb. 7, 2010)
- From Fish to Infinity (Jan. 31, 2010)

PREVIOUS POST: Harry Reid's Bipartisan Oblivion

NEXT POST: Home Fires: Women's Work

Steven Strogatz is the Schurman Professor of applied mathematics at Cornell University. Among his honors are MIT's highest teaching prize, membership in the American Academy of Arts and Sciences, and a lifetime achievement award for communication of math to the general public, awarded by the four major American mathematical societies. A frequent guest on National Public Radio's "Radiolab," he is the author, most recently, of "The Joy of  $\pi$ ," which grew out of his previous Opinionator series, "The Elements of Math." He lives with his wife and two daughters in Ithaca, N.Y. Follow him on Twitter @stevestrogatz

INSIDE OPINIONATOR

February 24, 2015

**COUCH** When the Best Sex Is Extramarital

Many people struggle to integrate love and lust in a single relationship. Read more...

February 14, 2015

**A Curious Case of Writer's Block**

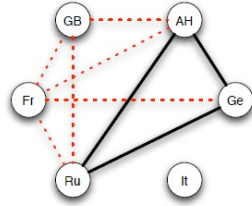
Could I help the patient with his problem in just a single session? Read more...

MORE CONTRIBUTORS

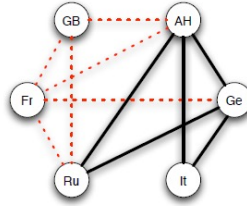
ARCHIVE Select Month

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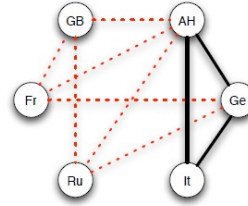
## Evolution of international relationship



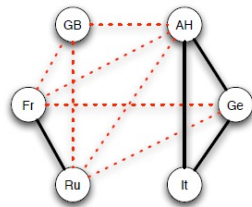
(a) *Three Emperors' League 1872-81*



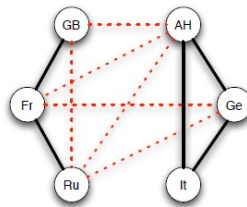
(b) *Triple Alliance 1882*



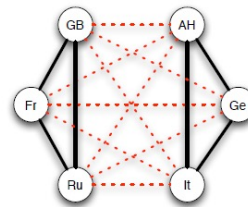
(c) *German-Russian Lapse 1890*



(d) *French-Russian Alliance 1891-94*



(e) *Entente Cordiale 1904*



(f) *British Russian Alliance 1907*

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## Agenda

- Network of friends and enemies
  - Balanced/stable configurations
- Local (a few nodes) → global (whole network)

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## Founders

- Fritz Heider (1940s)
- Cartwright and Harary (1950s)

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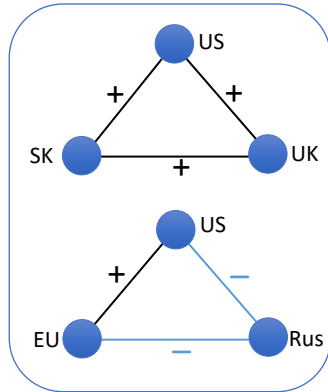
## First model (to be relaxed later)

- **Complete graph** (edge between each pair of nodes)
- Each edge: either + or –
- Example: sports team, countries, etc.

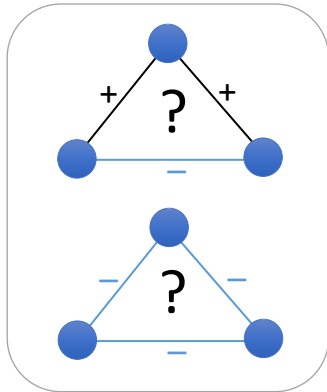
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# Def. Balanced triangle

Graph with just 3 nodes



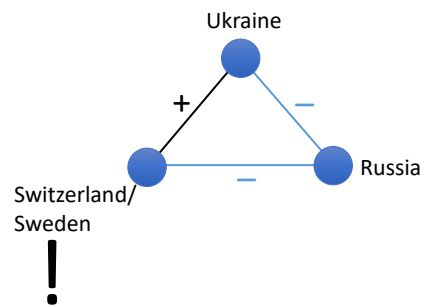
Balanced



Unbalanced

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# More examples (January 2022)



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## Def. Balanced complete graph

A complete graph is balanced  
if and only if  
every triangle in it is balanced

- Q1. Draw a balanced and an unbalanced complete graph.
- Q2. Is this definition a little extreme?

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## Characterization of balanced complete graphs

Balance Theorem (Harary, 1953)

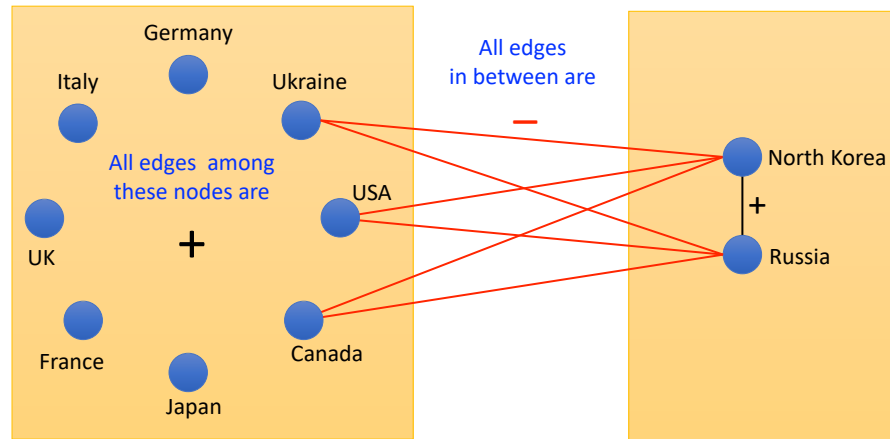
A labeled complete graph is balanced  
if and only if  
either (i) all edges are + or  
(ii) graph consists of two battling factions

**Local** (triangle) →

**Global** (either everyone gets along or there are two battling factions)

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## Example (January 2022)



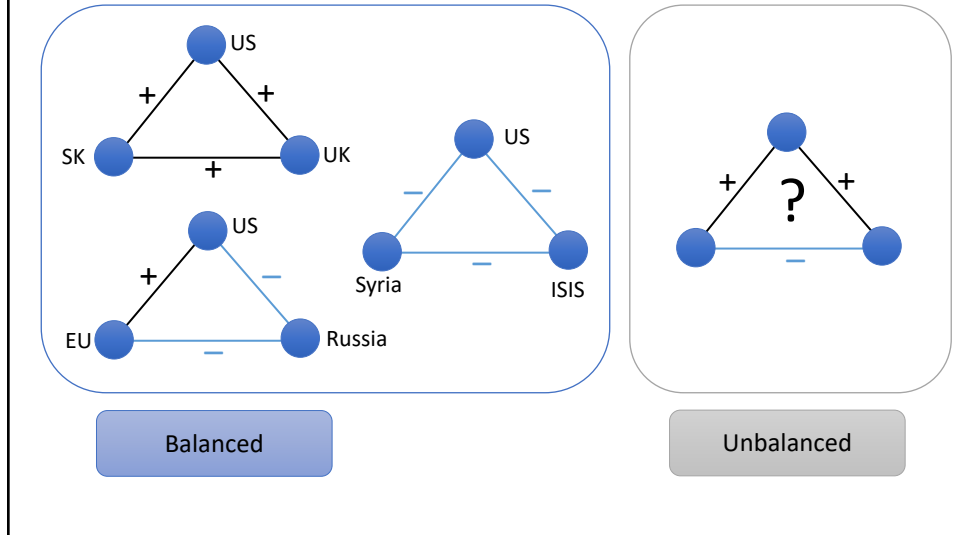
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## Extensions

1. Relax strict definition of balance
  - Weakly balanced complete graphs
2. General graphs (non-complete)

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## Ext.# 1: Weakly balanced networks



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## Characterization of weak balance

- Complete graph is weakly balanced if and only if either (i) all + or (ii) multiple battling factions
- Proof. Very similar to the previous characterization (with strict def. of balance)

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## Ext.# 2: General graphs

AKA **signed graph**

- may not be a complete graph!

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## Def: Balanced signed graph

A general signed graph is called balanced if it consists of two battling factions

- Definition is not based on triangles

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## Characterization: balanced signed graph

- A signed graph is balanced if and only if there exists no cycle with odd number of negative edges
- Q. What goes wrong otherwise?

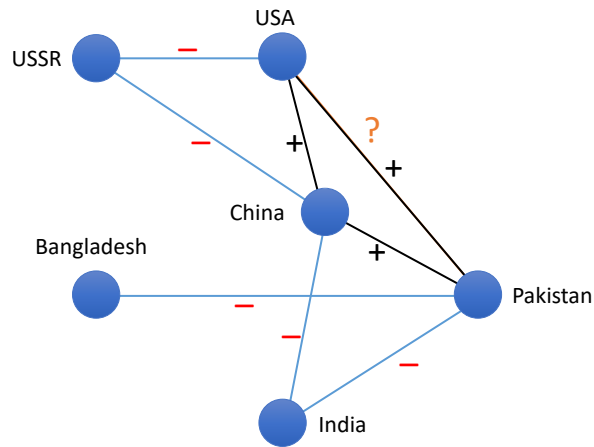
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Real-world examples

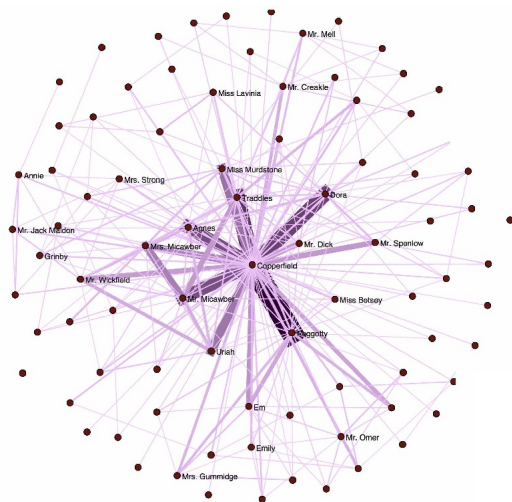
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## Liberation of Bangladesh in 1971 (Michael Moore, Journal of Social Psychology, 1978)



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## Social network among characters Graham Sack, 2013



### (b) David Copperfield (Charles Dickens)

- Nodes: 83
- Graph Density: 0.07
- Clustering Coeff: 0.80
- Avg. Degree: 5.81
- Stdev. Degree: 9.05
- Stdev / Avg. Degree: 1.56
- Avg. Shortest Path: 2.03
- Diameter: 4

AI question: generate narrative from the evolution of stability

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